A Slab Surface Energy Balance Model (SUEB) and Its Application to the Study on the Role of Roughness Length in Forming an Urban Heat Island

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Abstract. In the paper the simple slab urban energy balance (SUEB) model has been presented. The model consists of two sub-models: surface processes parameterizations and a simple boundary layer model. In the model, a town is treated as a single entity with specified physical parameters. The model has been tested while using the data available and the investigation into the role of roughness length in the urban heat island (UHI) has been presented.

Key words: urban climate, modelling, roughness length.

1. Introduction

The evolution of a planetary boundary layer is in any place determined by the surface energy balance. In extensive urbanized areas energy exchange between surface and the atmosphere is altered by many factors. Among these the most often mentioned are: an increased absorption of a short-wave radiation and a decreased long-wave radiation loss (canyon geometry – multiple reflection and reduced sky view factor), an increased long-wave radiation from the sky (air pollution), an increased heat storage (thermal admittance of materials), a decreased evapotranspiration (build-up area), anthropogenic heat source (building and traffic heat loss), and altered turbulent heat transport (high roughness length) (Oke, 1982). Measurements of the energy balance in an urban area are quite rare (Fortuniak et al., 2001) and provide complex information on the town energy balance rather
than on the role of each of the specified factors. Thus, numerical simulations of
the processes in the urban canopy layer (e.g. Oke et al., 1991; Masson, 2000) and
the urban boundary layer (e.g. Yu and Wagner, 1975) have been provided. The
main goal of this work is to present a simple ‘slab’ urban energy model (SUEB)
and to show its applicability to testing the influence of the specified parameters,
which change urban energy balance, on the urban heat phenomena.

In the slab models, a surface canopy layer has been treated as a single entity
with specified physical parameters. Neither singularities of the surface geometry
nor physical processes within the canopy layer have been considered. Models of
this type are relatively simple and the simplicity of some of their aspects may be
regarded as the advantage. First of all, because of a reasonable time of computa-
tion, it is possible to make many numerical experiments using a simple PC ma-
chine. These experiments allow testing the model’s sensitivity to different bound-
ary and initial conditions, as well as, they verify different parameterizations. In
addition, it is easier to understand the role of each factor in a simple model rather
than in a very complex one with a large number of interactions.

2. The Model Description

The surface energy balance of any place may be typically written as:

\[ Q^* = Q_H + Q_{LE} + Q_G \]  

(1)

where \( Q^* \) is the net all-wave radiation, \( Q_H \) and \( Q_{LE} \) are the turbulent sensible and
latent heat fluxes respectively, \( Q_G \) is the heat flux to the ground. In the system with
complex geometry an additional term, \( \Delta Q_S \), for the energy stored in a canopy layer
should be added. However, in the slab model which views a city as a single entity,
the energy stored might be simplified and treated as the heat accumulated in the
ground (a thick slab representing an urban canopy); here equation (1) is applica-
able. In the proposed model the components of the energy balance are calcul-
ated with the methods specified below.

2.1. Surface Radiation Budget

The surface radiation budget is determined by the incoming shortwave (\( I_{sob} \))
and long-wave (\( L_\downarrow \)) radiation, the surface radiation parameters (albedo – \( \alpha \), and
emissivity – \( \varepsilon \)) and the surface temperature \( T_s \):

\[ Q^* = (1-\alpha) I_{sob} + \varepsilon L_\downarrow - \varepsilon \sigma T_s^4 \]  

(2)

(\( \sigma \) is the Stefan–Boltzmann constant). In the proposed model a diurnal variation of
\( I_{sob} \) and \( L_\downarrow \) might either be obtained from measurements or modelled with the aid
of various formulas. However, to test the role of the factor steering urban energy
balance it is more convenient to use some numerical formulas rather than empirical
data. In the simulations presented in the article the short-wave radiation on the horizontal surface has been calculated with the method presented by Davis et al. (1975):

\[ I_{	ext{toth}} = S_0 \sin h_0 \tau_{\text{oa}} \tau_{\text{ob}} (1 + \tau_{\text{oa}} \tau_{\text{ob}} \tau_{\text{ob}})/2, \]

where \( S_0 \) is a solar constant, \( h_0 \) is a solar height, and \( \tau \) are transmissions due to water vapour absorption, aerosol absorption, water vapour scattering, aerosol scattering, and Rayleigh scattering. The formula has been tested having taken into account the data from \( \text{Lodz} \) and it showed the agreement with the measurements taken on sunny days. The incoming long-wave radiation \( L_\downarrow \) is commonly set to be a constant of the order of the daily average of measured values or it is calculated from empirical formula. Applicability of various empirical formulas (e.g. Idso and Jackson, 1969; Prata, 1996) has been tested, and it has been found out that, in general, they represent the daily average value of \( L_\downarrow \) quite well, in contrast to the diurnal variations then this representation is worse.

### 2.2. Heat Flux to the Ground

Heat flux to the ground is found by solving one-dimensional heat diffusion equation:

\[ \frac{\partial T}{\partial t} = \nu \frac{\partial^2 T}{\partial z^2}, \]

where \( \nu \) is thermal diffusivity. Equation (4) is solved numerically with the aid of Crank-Nicholson scheme for 10 levels. Bottom boundary conditions are set by a constant temperature at 1 m depth. The procedure to calculate the ground profile of temperature has been tested on an analytical solution, other models outputs and measurements (Fortuniak, 2001).

### 2.3. Turbulent Heat Fluxes in the Surface Layer

Determining the turbulent heat fluxes is one of the most difficult problems in modelling surface layer processes. A very popular parameterization based on the Monin-Obukhov similarity theory with the use of Businger’s functions for the flux-profile relationships comes from the work of Louis (1979). This method assumes an equal roughness length for momentum, \( z_{\text{om}} \), and heat transfer, \( z_{\text{oh}} \), whereas many measurements show a wide range of variation for \( z_{\text{om}}/z_{\text{oh}} \) ratio. In an urban area this ratio can even have the value of \( 10^2 \)–\( 10^3 \) or more (Voogt and Grimmond, 2000). A slight modification of Louis’s work considering a different roughness length for heat and momentum was proposed by Mascaro et al. (1995). Surface turbulent fluxes may be found to be derived from integrated flux profile relationships:
\[ u(z) = \frac{u^*}{k} \left[ \ln \left( \frac{z}{z_{0m}} \right) - \Psi_m \left( \frac{z}{L} \right) + \Psi_m \left( \frac{z_{0m}}{L} \right) \right], \]  \tag{5}

\[ \Delta \theta = R \frac{\theta_s}{k} \left[ \ln \left( \frac{z}{z_{0h}} \right) - \Psi_h \left( \frac{z}{L} \right) + \Psi_h \left( \frac{z_{0h}}{L} \right) \right], \]  \tag{6}

where \( u^* \) is the friction velocity, \( u \) – wind at height \( z \), \( \Delta \theta \) – potential temperature difference between heights \( z \) and \( z_{0m} \), \( \theta_s \) – scale temperature for heat flux, \( R \) – neutral turbulent Prandtl number, \( k \) – von Karman constant, \( L \) – Obukhov length. The stability functions \( \Psi_m \) and \( \Psi_h \) have the typical form. Using the bulk Richardson number at the height \( z \):

\[ Ri_b = \frac{g \Delta \theta}{\theta u^*}, \]  \tag{7}

the stability parameter \( z/L \) may be expressed as a function of \( Ri_b \) and roughness lengths:

\[ z = \left( \frac{Ri_b}{R} \right) \left[ \ln \left( \frac{z}{z_{0m}} \right) - \Psi_m \left( \frac{z}{L} \right) + \Psi_m \left( \frac{z_{0m}}{L} \right) \right] \left[ \ln \left( \frac{z}{z_{0h}} \right) - \Psi_h \left( \frac{z}{L} \right) + \Psi_h \left( \frac{z_{0h}}{L} \right) \right]. \]  \tag{8}

An iterative solution of (8) allows to calculate \( u^* \) and \( \theta_s \) (from Eq. 5) and, in consequence, the turbulent heat and momentum fluxes (kinematic) in the surface layer:

\[ \overline{w'}u' = u^*_u, \quad \overline{w'}\theta' = u^*_\theta. \]  \tag{9}

To determine a latent heat flux, an aerodynamic resistance concept has been applied. For a heat flux an aerodynamic resistance is given by: \( r = \Delta \theta / \overline{w'\theta'} \). In calculations of the turbulent moisture flux one should consider an additional surface resistance, \( r_s \):

\[ \overline{w'}q' = (q_{sat}(T_s) - q)/(r + r_s), \]  \tag{10}

where \( q_{sat}(T_s) \) is the saturated specific humidity at the temperature \( T_s \) and \( q \) is the atmospheric specific humidity at the level \( z \). Surface resistance has been calculated by using the procedure given by Best (1998) with a slight modification of the surface water evaporation.

### 2.4. Boundary Layer Model

To be able to apply Eqs (5)–(10) some information about the wind, temperature and specific humidity at the level \( z \) is necessary. In general, it can be derived
either from measurements or from other models. For this purpose a relatively simple one-dimensional, first-order model with $K$-closure technique for 28 levels (up to 5 km) has been constructed. Even if the model is one-dimensional, a local advection may be estimated by providing a simultaneous estimation for 'rural' and 'urban' sites located at some arbitrarily chosen distance. In the model different local turbulence closures schemes have been tested: Louis (1979), Mellor and Yamada (1982), Gambo (1978), Sievers and Zdunkowski (1986). The comparison of the modelled temperature profiles with the measurements on the 33$^{rd}$ day of Wangara experiment (Fig. 1) shows that all the parameterizations represent

![Graphs showing modelled temperature profiles](image)

Fig. 1. Modelled (dashed) temperature profiles for different turbulence parameterizations versus measured (solid) values on the 33$^{rd}$ day of Wangara experiment
a diurnal evolution of temperature in a boundary layer similarly well. Thus, because of a relatively high numerical stability, the Louis’s method (1979) has been used in the majority of further simulations.

3. Preliminary Results

A simulation on 2 May 2001 is an example of the application of the model to the study on the urban energy balance (Fig. 2). The measured energy fluxes for the urban site (Fig. 2A) come from the urban energy balance system functioning in Łódź. The system had been developed at Indiana University by Sue Grimmond and Brian Offerle and was installed in Łódź in November 2000 (Fortuniak et al., 2001). Sensors have been fixed on the top of a mast on the roof of a five-storey building at the height which exceeds the roof level approximately two times. Because of such elevation the measured fluxes not only represent a value typical for a specified surface (i.e. roof, street, lawn), but also a spatial average for a relatively large part of the town (Offerle et al., 2002). The analysed day was characterized by favourable weather conditions with a slight wind and an almost cloudless sky. The radiation budget has a typical diurnal variation with a maximum at noon (600 W · m⁻²) and negative values during the night (reaching a minimum after sunset – about −100 W · m⁻²). The incoming longwave radiation has no clear diurnal variation but it decreases roughly linearly during 24 hours from 340 W · m⁻² to 280 W · m⁻². In the simulations mentioned above the mean value of measured \( L_\downarrow \) (310 W · m⁻²) was taken. Turbulent heat flux is almost zero during the night and increases regularly reaching in the afternoon about 350 W · m⁻². Latent heat flux has quite an irregular diurnal pattern, but neglecting some effects of dew on a krypton hygrometer which probably affect the morning measurements, it varies from nearly zero values at night to about 100 W · m⁻² in the afternoon. Heat stored in the canopy layer is calculated as a residual part from the energy balance and is characterized by negative values during the night and morning hours (up to −150 W · m⁻²) and maximum before noon (230 W · m⁻²). Modelled fluxes (Fig. 2B) show a good agreement with the observed diurnal variation of the urban energy balance components, especially of \( Q^* \) and \( Q_h \). In case of \( Q_{ke} \), the values measured in the morning hours turned out to be much lower. The least precise estimation has been obtained for the stored heat, which in the model is represented as heat flux to the ground. Modelled values are about 50 W · m⁻² lower than measurements but, in general, a diurnal variation of the parameter is well represented. This only proves that the presented model can represent the real urban energy balance. An additional information on other parameters is necessary to provide a reliable validation of the model. In general, such information is not available from measurements and must be assumed arbitrary. In the present simulation these parameters having been based on values quoted in literature are approximate. Emissivity is assumed to be the same for urban and rural sites (\( \varepsilon = 0.9 \)). Other parameters differ
for both places: albedo ($\alpha_U = 0.08$ - equal mean measured value, $\alpha_R = 0.20$), thermal admittance ($\nu_{dU} = 1500 \, \text{J} \cdot \text{m}^{-2} \cdot \text{S}^{1/2} \cdot \text{K}^{-1}$, $\nu_{dR} = 748 \, \text{J} \cdot \text{m}^{-2} \cdot \text{S}^{1/2} \cdot \text{K}^{-1}$), roughness length for momentum ($z_{0mU} = 0.6 \, \text{m}$ - calculated by Klysik (1998), $z_{0mR} = 0.1 \, \text{m}$), roughness length for temperature ($z_{0thU} = 0.00001 \, \text{m}$, $z_{0thR} = 0.01 \, \text{m}$), soil moisture content ($sm_U = 0.05 \, \text{g/g}$, $sm_R = 0.25 \, \text{g/g}$). Upper wind is taken $6 \, \text{m} \cdot \text{s}^{-1}$ which gives a daily course of the wind at 10 m height similar to the measured one.

There are no measurements for the rural station but simulated fluxes (Fig. 2C) provide a typical picture for urban and rural environments. In the rural case, due to ground wetness, latent heat significantly contributes to the total balance, whereas in a city it is only slightly pronounced.

Fig. 2. Measured (A) and simulated (B) energy balance for the urban site and simulated energy balance for the rural site (C) on 2 May 2001.
The model allows to test a separate influence of the specified parameter on singularities of the urban climate, e.g., intensity of the urban heat island. A good example is the role of the roughness length in the process of formation of nighttime urban-rural thermal contrasts. In the presented simulations (Fig. 3) the analysed cases differ only in roughness length. Other parameters stay unchanged – they are fixed similarly to those in the above example. Figure 3A shows that a roughness length for temperature \( z_{th} \) plays a very important role in forming nighttime urban heat island. For the places which significantly differ in \( z_{th} \) (e.g., \( z_{th} = 0.1 \) m and \( z_{th} = 10^{-6} \) m), thermal contrasts during the night can reach a few degrees (3°C in the analysed case). The roughness length for momentum \( (z_{om}) \) has a minor meaning (Fig. 3B). When both roughness lengths are equal to each other \( (z_{th} = z_{om}) \), changes of this parameter do not lead to the well distinguished UHI at night. Thus, in the numerical simulations of the atmosphere’s evolution over urbanized areas, different values of the roughness length for momentum and temperature should be considered. However, in spite of \( z_{om} \), in case of thermal roughness length there is no simple method to estimate it on the basis of geometrical properties of a town. Moreover, extremely small values of \( z_{om} \) for urban areas (sometimes even smaller than the size of a single atom!) which can be found in literature cause problems with its physical interpretation.

![Graph A](image1.png)

![Graph B](image2.png)

Fig. 3. Influence of the roughness length on a daily temperature course: A – altered roughness length for temperature with other parameters being constant, B – altered roughness length for momentum with other parameters being constant

4. Conclusions

The simulations presented in the paper show that even simple numerical models, easily conducted on standard PC computers, can be a useful tool for the study on the urban climate. The main goal of such models is not to provide very accurate
meteorological forecasts but to help to understand processes which distinguish an urban environment from the natural one. In the paper the role of thermal roughness length in creating urban heat island phenomena is proved to be significant. Together with thermal properties of building materials, optical properties of a town and its moisture availability, the roughness lengths compose a set of steering parameters of urban energy balance. Those parameters complemented by meteorological factors, mainly by wind speed and cloudiness allow to solve a wide range of urban climate problems. For example, simulations with different parameters might help to find functional dependences of UHI on meteorological factors – in constructing statistical models. Parameters of these functions can be later fit using standard statistical methods. Nevertheless one must keep in mind restrictions of the models based on simple K-closure methods and Monin–Obukhov similarity, especially under weak wind conditions.

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References

Bryłowy model bilansu energetycznego (SUEB) i jego zastosowanie do badań roli współczynnika szorstkości aerodynamicznej w powstawaniu zjawiska miejskiej wyspy ciepła

Streszczenie

W opracowaniu przedstawiono bryłowy model bilansu energetycznego miasta. Model ten składa się z dwóch podmodeli: parametryzacji procesów powierzchniowych i jednowymiarowego modelu warstwy granicznej. Miasto traktowane jest w modelu jako jednolita bryła o specyficznych właściwościach fizycznych. Oprócz weryfikacji modelu przedstawiono również jego zastosowanie do badań roli współczynnika szorstkości aerodynamicznej w powstawaniu zjawiska miejskiej wyspy ciepła.

Słowa kluczowe: klimat miasta, modelowanie, współczynnik szorstkości.